Maths in Industry in Australia – Early Days

Graeme Hocking

Mathematics & Statistics
Murdoch University

ICIAM July 2019

My first .... 1986 ANZIAM Conference

APPLIED MATHS CONFERENCE 1986

MATHEMATICAL PROBLEMS IN INDUSTRY

In the last 2 years successful and interesting conferences on Mathematical problems in industry have been held in Melbourne and Sydney.

This year it has been decided to present several problems in similar format at the Applied Maths Conference.

Four problems will be described at the conference on the morning of Monday 10th. Later in the conference several sessions have been designated for further consideration of these problems. On the final morning of the conference a summary of progress made on each problem and suggestions for further work will be presented.

It is hoped that these problems will attract wide interest and attention during the conference.

A preliminary description of the 4 problems is attached to allow those interested to come partly prepared if they wish.

B.R. BENJAMIN

24th January, 1986.

The Schedule

MONDAY, 10 FEBRUARY - P.M.

	ROOM 1	ROOM 2	ROOM 3
1.50-2.15	RYAN: A Learning Curve Optimization Model for Plant Commissioning	GRIMSHAW: Extreme Inter-facial Waves	MILLER: Timestepping a Non-Linear Heat Equation
2.15-2.40	EUSTACE*: Discrete Dynamic Model for a Robot	HOCKING: Critical with-drawal from a two-layer fluid	NOYE: Solving Two-Dimensional Diffusion Problems using Boundary Element, Finite Element and Finite Difference Methods
2.40-3.05	LEE: Scheduling a Rail Tank Fleet	GRUNDY*: Waves on a static water surface beneath a layer of moving air	JACKETT: A Numerical Model of the Leeuwin Current: An application of the method of Lines
3.05-3.30	WYNTER: Elliptic Ortho-gonal Polynomials	STOKES: Sound Generated by Vortices Passing Aerofoils	HARMAN: Numerical approxima-tion and geometric differences
3.30-3.50	Afternoon Tea		
	INVITED LECTURES: ROOM 2		
3.50-4.30	HINCH: Micro-Models for two phase flow through a porous medium.		
4.30-5.10	KATZ: Biophysical Mechanisms in Sperm Transport and Fertilization.		
5.10-7.30	B-B-Q at Oval or Dinner in Restaurant		
7.30-9.00	ROOM 1: Monitoring of Gas Injection in Petroleum Reservoirs ROOM 2: Lot Sizing in a Materials Requirements Planning System ROOM 3: Longitudinal Surge in Road Tanker Design COMMITTEE ROOM: Automatic Pattern Recognition in Distress Communication Channels.		

* Denotes Student Paper

The Problems

AUTOMATIC PATTERN RECOGNITION IN DISTRESS
COMMUNICATION CHANNELS

If the following interesting problem in pattern recognition could be solved then a new and potentially useful device could be built which would provide for an automatic monitor of emergency distress channels. While this need has arisen from a specific area of interest any solutions to this problem could also have general application to a wide range of difficult problem areas and could lead to other fruitful recognition devices.

LONGITUDINAL "SURGE" IN ROAD TANKER DESIGN

Jarmyn Engineering Pty Ltd make road tankers for liquids transport. Road regulations impose weight limits on the gross load carried by groups of axles in a tanker-prime mover combination and on the gross weight of the loaded vehicle. To remain competitive, therefore, Jarmyn must ensure that its products are as light as possible, so that their customers - companies involved in liquids transport - can maximise their payloads and hence profitability.

Tankers are typically used to carry a number of different liquids with varying densities; as an extreme example, some fuel companies will carry liquids with specific gravities ranging from 0.73 to 1.65. Tankers are usually sized so that a full load of the lightest liquid approaches the limit set by road authority load regulations; when carrying denser liquids, therefore, they must be partly empty.

LOT SIZING IN A MATERIALS REQUIREMENTS PLANNING SYSTEM

1 Introduction

This problem considers an inventory system in which end products are assembled from components, which are purchased items or made from raw materials. Only the end products have demands, which are assumed to be random. A materials requirements planning (MRP) system determines the lot sizing of orders and the scheduling of production under the assumption of known demands. What is a good policy for reducing costs when operating an MRP system with stochastic demands?

MONITORING OF GAS INJECTION IN PETROLEUM RESERVOIRS

The injection of gas into petroleum reservoirs is often employed to maintain reservoir pressures and to sweep hydrocarbons towards production wells. Production and injection wells are usually drilled in a geometric pattern (such as shown in Figure 1) to give optimal displacement efficiency.

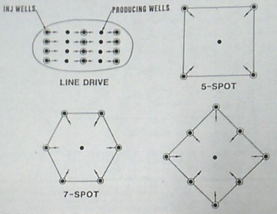


FIGURE 1 : PRODUCTION/INJECTION WELL PATTERNS

The success of a gas injection scheme depends largely on the degree of reservoir heterogeneity. Quite often the hydrocarbon reservoir is composed of a number of individual layers of differing properties. The gas introduced at the injection well will tend to favour entry into the most permeable of the layers (see Figure 2). This preferential movement of gas can be detrimental to ultimate recovery since much of the hydrocarbon reserves would be by-passed by the injection gas. Lateral variation in reservoir quality also reduces recovery by decreasing the horizontal displacement efficiency.

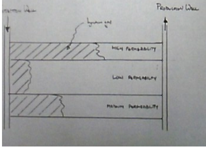


FIGURE 2 : CROSS SECTION OF LAYERED PETROLEUM RESERVOIR WITH GAS INJECTION.

Standalone Study Groups - $\approx$ 3 year cycle

- Bob Anderssen, Frank de Hoog, Noel Barton at CSIRO in Canberra
- Kerry Landman, Noel Barton, University of Melbourne
- Sean McElwain, Queensland University of Technology, Brisbane
- Phil Howlett, University of South Australia
- Tim Marchant, Maureen Edwards, Geoff Mercer, University of Wollongong
- Graeme Wake et al, Massey University, New Zealand.
- John Shepherd, Royal Melbourne Institute of Technology
- Troy Farrell et al, Queensland University of Technology, Brisbane
- Peter Pudney, University of South Australia
- Natalie Thamwattana, University of Newcastle

Standalone Study Groups - Their own identity

Organizers always tried to get a diverse range of problems ....

- Melbourne, Adelaide (2) – business/commercial/statistical/financial
- New Zealand – primary industry, agriculture, NZ steel
- Brisbane quite diverse – power networks, medical devices, water pumping, small industry
- Wollongong – encryption, power grid competition (financial maths), steel industry, climate change
- Adelaide (1) – trains, wine, biscuits, light industry

Interesting Problems

- Analysis of Train Wheel Noise
- Pressure drop in pipelines due to pump trip event
- Flow of non-Newtonian fluids in open channels
- Modelling microbial pollutant loads associated with surface water run-off in water supply catchments
- Coating deformations in the continuous hot dipped gavanizing process
- Optimisation of an ultrasonic nebulizer
- Cavity formation and entrainment in deep submerged waterjets
- Implementing Lanier's patents for stable, safe economical ultra-short windg vacu- and para-planes
- Tsunami risk modelling for Australia: understanding impact of data
- Math models for uptake of agrichemicals through plant leaves

Brief Aside – South African Study Groups

- David Mason (ably assisted by Ashleigh Hutchinson) from the University of Witswatersrand, Johannesburg, South Africa has run study groups for the last 12(?) years.
- Generally alternated between U. Wits and AIMS (African Institute for Mathematical Sciences), Cape Town.
- Strong African flavour, from mining to environment to commercial.
- A very strong training component for African students, with many more students than research academics.
- Mine collapse, Johannesburg bus system, user agent strings, car parking optimisation, water seepage in mines, automatic pattern recognition, exploding lakes, rogue waves, extreme swimming water fins, curve on a football at altitude, never-ending sugar manufacture problems ....
- I recommend it as an interesting exercise in a different environment.

Does a study group ever fail?

- Most study groups provide the industry partner with at least a different perspective.
- I've only ever seen ONE industry partner complain openly.
- Gains range from complete solution of the problem (often in OR or optimisation) $\longrightarrow$ significant improvement in process $\longrightarrow$ better understanding $\longrightarrow$ new way forward.
- However, sometimes WE may feel we could have done more, or perhaps just get an interesting problem for further work .....
- Here are a few from Australia/New Zealand study groups.

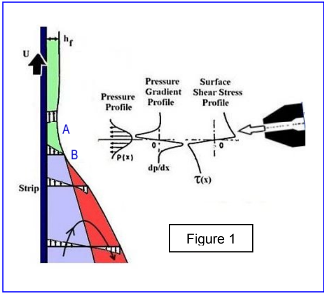
Most interesting "failures"

- Wound pipe "swimmers" ....
  - Metal wrapped around "swimmers" at high speed to make pipe.
  - Size of the "swimmers" determined interior pipe radius.
  - "Swimmers" failed (broke) irregularly, but quite often (expensive).
  - No data (Engineer fired for suggesting it)
  - Study Group ended up suggesting - collect the data (then come back - they never did!)
- Ultrasonic Nebulizer (Vlad the Inhaler)
  - Medical device to deliver drug through tube device - much easier.
  - Drug atomized with ultrasonic crystal.
  - Atomized particles too large to inhale effectively - make them smaller?
  - Particle size $\propto$ frequency of oscillation - crystal property.
  - None available at that time .... just wait?
  - This problem is REALLY interesting because of the atomization process.
  - On my to-do list of future work.

Continuous coating of steel - stripping with air-knife

- The problem pre-dates the Australian Study Groups
- I met it in my honours year .... at Adelaide University.
- Ernie Tuck was contacted by Port Kembla steel works (now Bluescope steel).
- Prediction of coating thickness during continuous coating process.
- Too hot to handle - or at least go in and see.
- Returned 2 times to MISG - Australia (Wollongong), Ireland (Dublin)
- One of my favourite examples of modelling fluids - I use it in teaching.

The problem



- Sheet passed through molten zinc alloy (2 m/s)
- Air jet (2 mm, 200 m/s) at B reduces coating thickness
- higher pressure $\Rightarrow$ thinner coat, higher speed $\Rightarrow$ thicker or thinner
- Most recent problem: Defects at high speed

Uni-directional flow - Exact Solution for gravitational flow.

- Viscous, incompressible fluid in a steady state.
- Sheet of metal is broad and flat $\Rightarrow$ two dimensions
  ..... could equally do it for a general cylinder.
- Assume unidirectional flow, but must keep gravity.
- Continuity equation $v = w = 0 \Rightarrow u_x = 0 \Rightarrow u = u(y, z)$
- The Navier-Stokes equations give,

$$\frac{D\mathbf{q}}{Dt} = -\frac{1}{\rho}\nabla p - g\mathbf{i} + \nu\nabla^2\mathbf{q}$$

$$\frac{\partial u}{\partial x} = 0 \Rightarrow 0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + \nu\left(u_{yy} + u_{zz}\right)$$

$$p'(y) = p'(z) = 0 \Rightarrow \qquad \nu\left(u_{yy} + u_{zz}\right) = g - p'(x)/\rho$$

Uni-directional flow - Exact Solution for gravitational flow.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v \Rightarrow p_y = 0$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \Rightarrow p_z = 0$$

$$\Rightarrow p(x) \text{ only}$$

$$u_x + u_y + w_z = 0 \Rightarrow \frac{\partial u}{\partial x} = 0 \Rightarrow u(x, y) \text{ only}$$

$$\Rightarrow \qquad \nu\left(u_{yy} + u_{zz}\right) = g - p'(x)/\rho$$

Boundary conditions

- At the liquid metal interface, the fluid sticks.
- Stress components on the surface are zero on outer edge.
- Pressure along free surface is constant, $p'(x) = 0$
- The full problem, invoking 2d assumption is

$$\frac{d^2u}{dx^2} = g/\nu$$

$$u = U, \;\; z = 0,$$

$$u_z = 0, \;\; z = h$$

- Integrating twice

$$u(z) = U + \frac{1}{2} z(z - 2h)\frac{g}{\nu}$$

Maximum Flux Condition

- This velocity profile valid for any h.
- h can be determined by the flow out of the bath
- Early work used the maximum flux criteria.

$$Q = \int_0^h u(z)dz \ = -\frac{g}{3\nu}h^3 + Uh$$

- To maximise Q, $\frac{dQ}{dh} = 0$

$$\Rightarrow -\frac{g}{\nu}h^2 + U = 0 \Rightarrow h^* = \sqrt{\frac{U\nu}{g}}.$$

- Speed on the free boundary $u(h^*) = U/2$
- Maximum flux given by $Q_{max} = \frac{2}{3}U^{2/3}(\nu/g)^{1/2}$.

Improved model - “mainly” vertical flow with air knife

Navier-Stokes and continuity equations with boundary conditions;

$$u = U, \quad w = 0 \quad \text{on } z = 0$$

$$\begin{cases} \mu u_z = \tau_a(x), & \\ p - p_a(x) = -\gamma\kappa, & \text{on } z = h(x,t) \\ h_t + u h_x = w. & \end{cases}$$

U speed, $z = h(x,t)$ is the layer thickness,
γ surface tension, $\kappa \sim h_{xx}$ curvature,
$p_a(x)$, $\tau_a(x)$ pressure, shear stress from air knife (assumed known).

"Final" PDE

After much analysis - assuming slowly varying etc. an equation for $h(x,t)$;

$$h_t + \left[h - \frac{S}{3} h^3 + \frac{1}{2} h^2 g(x) + \frac{1}{3} \left(\left(\frac{\epsilon^3}{Ca} \gamma(x) h_{xx} \right) - P^* p_a(x) \right)_x \right]_x = 0$$

S gravity (Stokes No.), Ca Capillary No., γ Surface tension,

P^* pressure parameter, $p_a(x)$ is air pressure,
$g(x) = \tau + \gamma'(x) + \phi$ where $\tau =$ air shear, and ϕ is some unknown effect due to oxide layer.

Have determined the order of magnitude of all of these terms to determine their importance.

and the conclusion is ...

The appropriate 1st-order, nonlinear advection pde is of the form

$$h_t + c(h, p_a', \tau)h_x = A(h, p_a'', \tau')$$

where $c(h, p_a', \tau) = 1 - (P^* p_a' + S)h^2 + h\tau$, is a speed.
and $A(h, p_a'', \tau') = \frac{1}{3}h^3 P^* p_a'' - \frac{1}{2}h^2\tau'$ is amplification.

With this rather unpleasant equation, we can

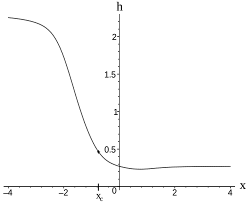
- Compute possible steady state solutions - then perturb it
- Compute characteristics etc. given $p(x), \tau(x)$ (known)
- Compute time evolution of an initial surface

Steady State solution

In a steady state, get a cubic for $h(x)$ at each point;

$$\text{Flux } Q = f(h,x) = h + \frac{h^2}{2}G(x) - \frac{h^3}{3}(S + P'(x))$$

Each different x gives different Q?!
Can maximize Q at each section x,
but they all have to join up.
Take the minimum of the maximum fluxes.....
..... BUT DOES IT WORK ....?
Industry people use it in real time.



Vertical characteristics

PDE theory gives vertical characterisitics if $c(h, p'_a, \tau) = 0$;

$$1 - (P^* p'_a + S)h^2 + h\tau = 0$$
$$\Rightarrow h_V = \frac{\tau}{2P^* p_a}\left(1 \pm \sqrt{1 + \frac{4P^* p'_a}{\tau^2}}\right)$$

This means;

if $h > h_V$ at any point, disturbances propagate backwards
if $h < h_V$ at any point, disturbances propagate forwards.

Interestingly, if no air-knife $h_V = \frac{1}{\sqrt{S}}$

(which coincides with the maximum flux solution under gravity).

Characteristic Traces

Characteristics obtained from

$$\frac{dh}{dx} = \frac{A(h, p_a'', \tau')}{c(h, p_a', \tau)}$$

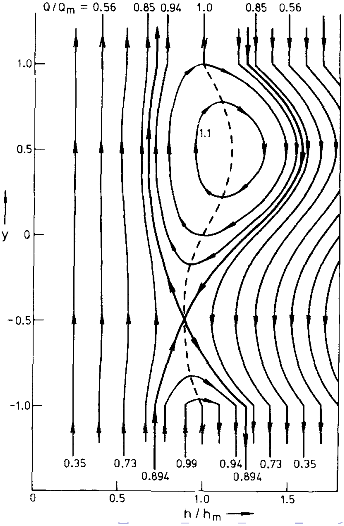
Given $p_a'(x), \tau(x)$ can compute trajectories in the $h - x$ plane.
These coincide with surface shapes given an initial $h = h_0$.
Only some of these will be feasible/valid.

Example trajectories

If give a "reasonable" (but solvable) $p_a'(x)$, ignoring τ, (see Tuck, 1983) i.e.

$$p_a'(x) = \begin{cases} 0; & x < -1 \\ -x - x^2; & -1 < x < 0 \\ -x + x^2; & 0 < x < 1 \\ 0; & x > 1 \end{cases}$$

analytical solution for trajectories
One coincides with the
vertical characteristic (upstream)
Arrows indicate direction of time ...



- Study Groups in Australia (and sometime NZ) have been generally successful
- Support from academics definitely depends on the nature of the problems
- Industry has had some spectacular successes in participation
- A number of industries continue to return
- NEXT: University of Newcastle, NSW - late January 2020.