Dynamical Systems in Origami/Kirigami Tessellations

Rinki Imada* and Tomohiro Tachi
Department of General Systems Studies, Graduate School of Arts and Sciences, The University of Tokyo

Research Topic: Nonperiodic Folding of Periodic Origami

Kinematics of Rigid Origami

- Kinematics of **rigid origami** has played a central role in origami science/engineering, where facets/creases of origami are replaced by rigid pabels/rotational hinges.
- The preservation of the shapes of panels and their connectivities impose multiple nonlinear constraints, which are generally hard to solve directly.

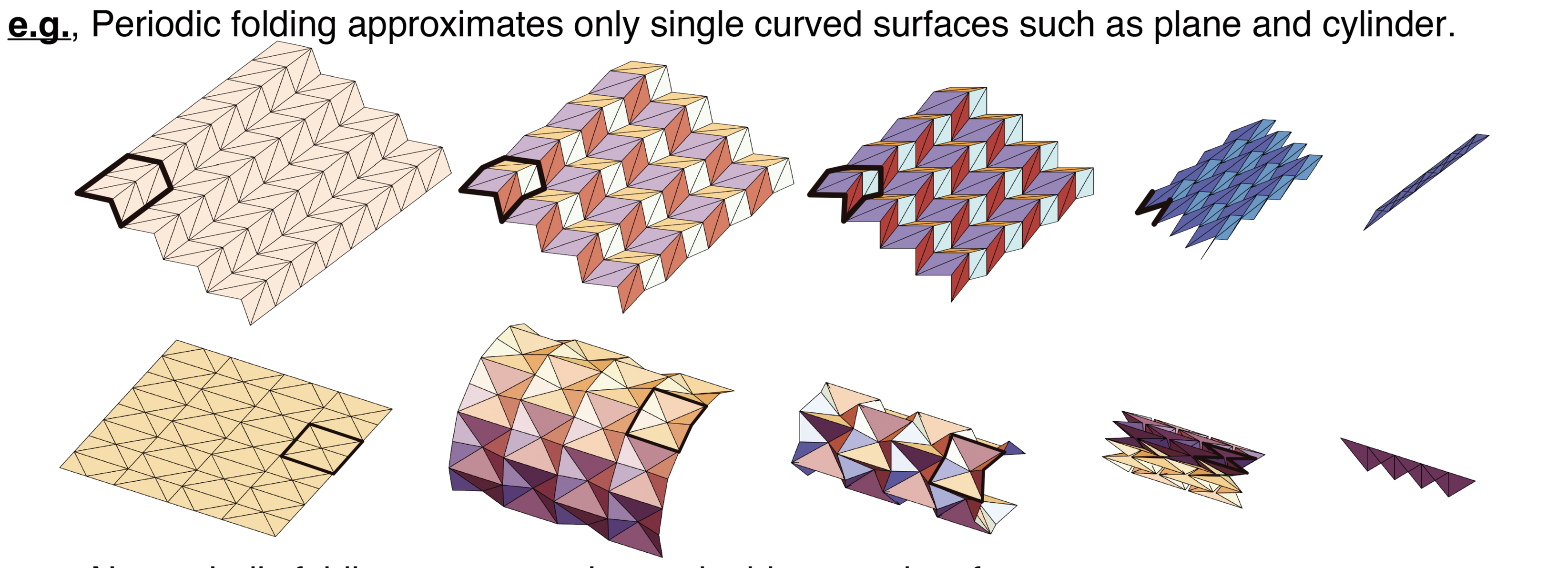
Periodic Folding of Periodic Origami

- The periodicity makes solving the kinematics easier and leads to **global deformations** of the entire structure, which is useful for engineering applications.
- However, it also limits the potential of periodic origami, i.e., **origami tessellations**.

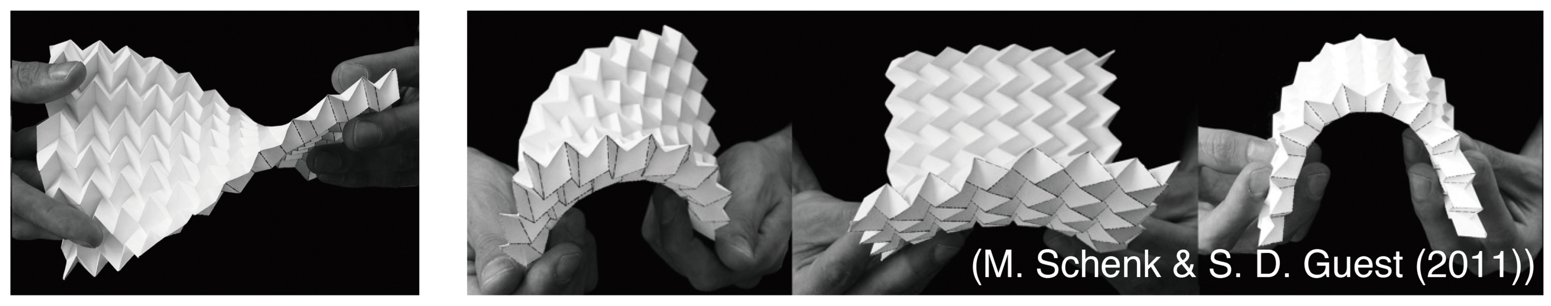
Nonperiodic Folding of Periodic Origami

- Although it is a source of interesting phenomena that cannot be feasible through periodic folding, it is hard to solve and mathematically understand the kinematics.
- Also, there can be not only global deformation but also **local deformations**.
- **<u>We established a novel mathematical model of nonperiodic folding, dynamical systems of origami tessellations, and found some nonlinear global deformations.</u>**

<u>e.g.</u>, Periodic folding approximates only single curved surfaces such as plane and cylinder.



<u>e.g.</u>, Nonperiodic folding can approximate doubly curved surfaces.



(M. Schenk & S. D. Guest (2011))

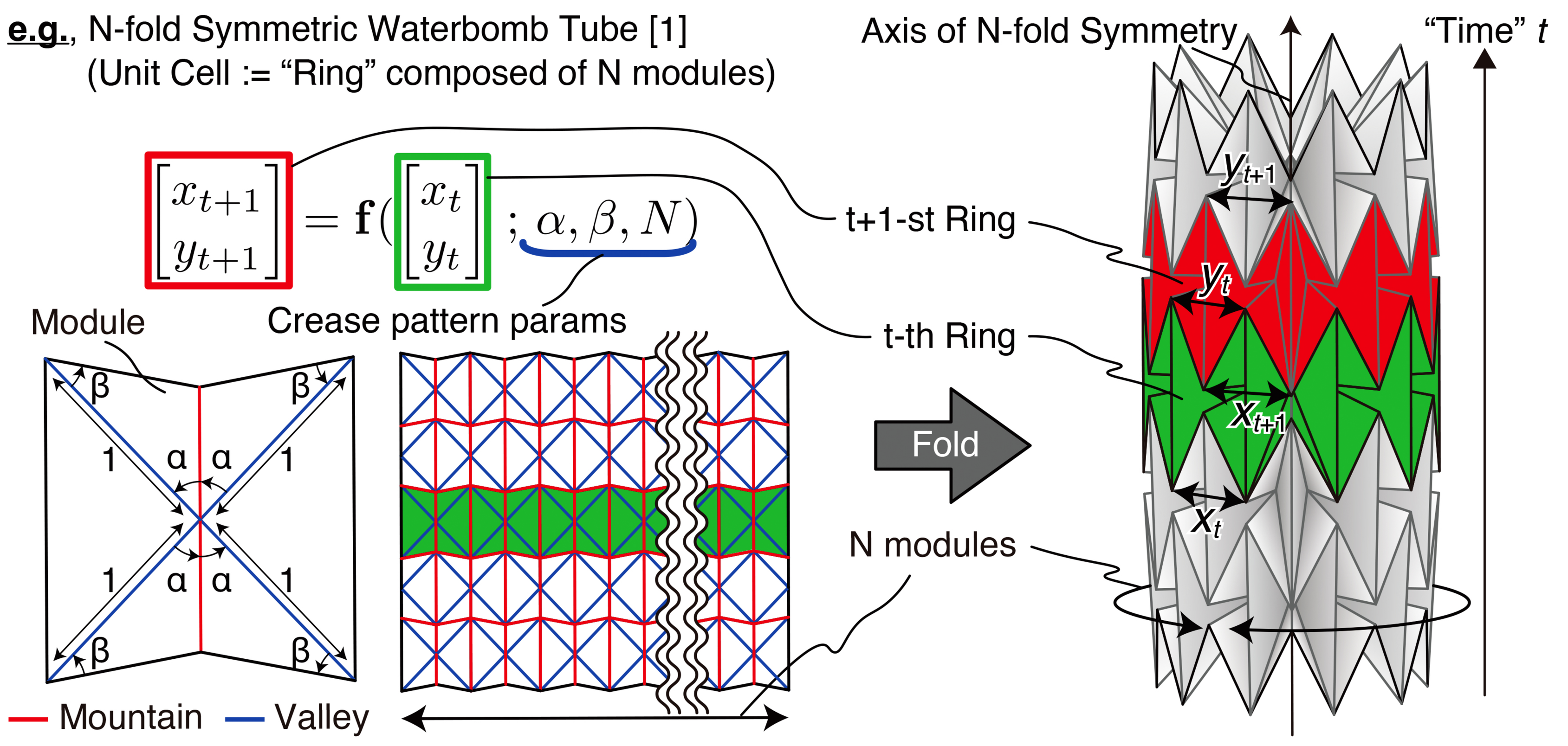
Proposed Model: Dynamical Systems of Tessellated Structures

Determinictic Origami Tessellation

- An Infinite sequence of **unit cells**, where the folded state of a unit cell determines that of its adjacent one because of the geometric constraints.
- We define the **discrete dynamical system F**: $\mathbf{x}_t \mapsto \mathbf{x}_{t+1}$, where $\mathbf{x}_t$ represents the folded state of t-th unit cell. Then, $\{\mathbf{x}_0, \mathbf{x}_1 ...\}$ represents the entire folded state.

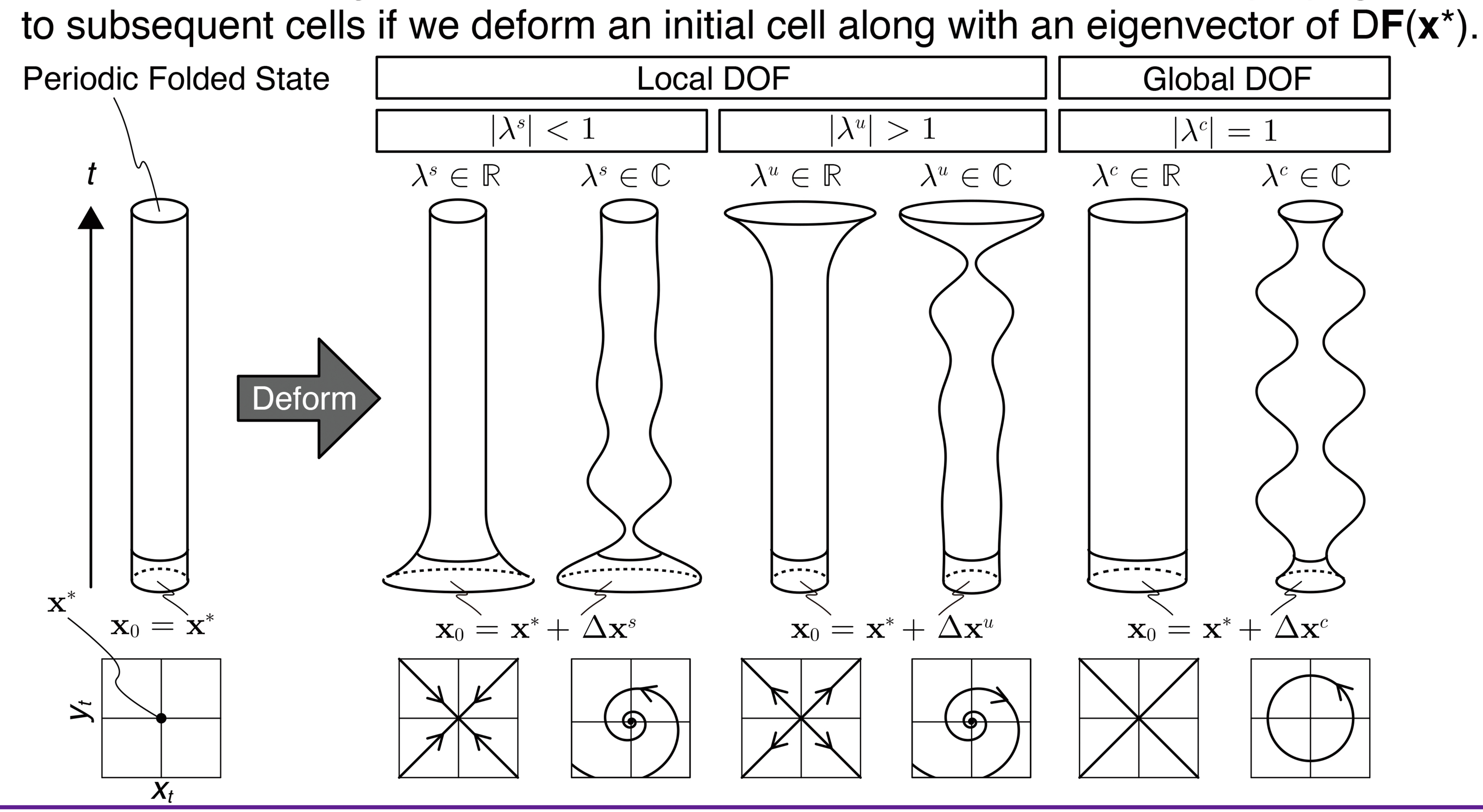
<u>e.g.</u>, N-fold Symmetric Waterbomb Tube [1]
(Unit Cell := "Ring" composed of N modules)

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = \mathbf{f}\left(\begin{bmatrix} x_t \\ y_t \end{bmatrix}; \alpha, \beta, N\right)$$



Global/Local DOF of Periodic Folded State and Linear Stability Analysis

- A **Fixed Point x*** satisfying **F**(**x***)=**x*** corresponds to a periodic folded state.
- The **Linear stablity** tells us how the deformation in an initial unit cell propagates to subsequent cells if we deform an initial cell along with an eigenvector of **DF**(**x***).

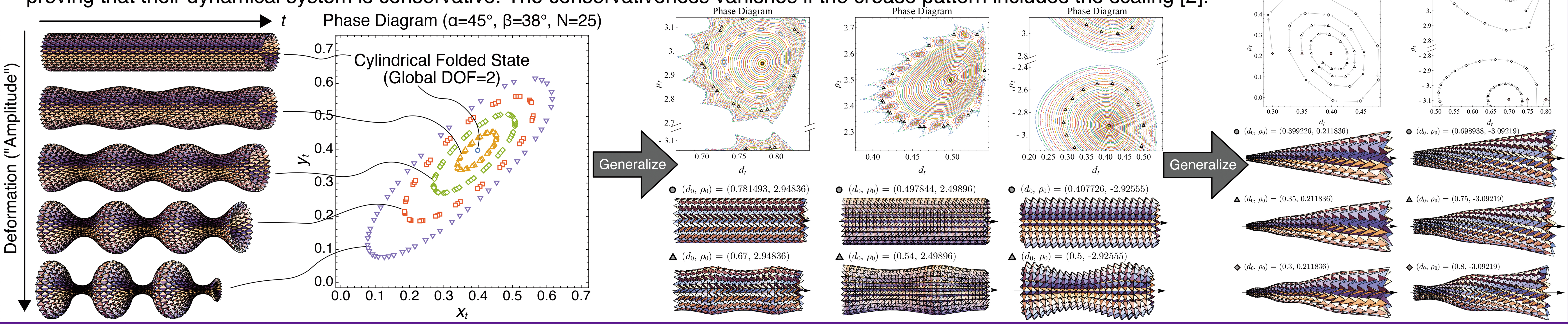


Result: Some Global Deformations in Nonperiodic Folding and Connection to Mathematical Structures

Global Deformations Induced by Global DOF

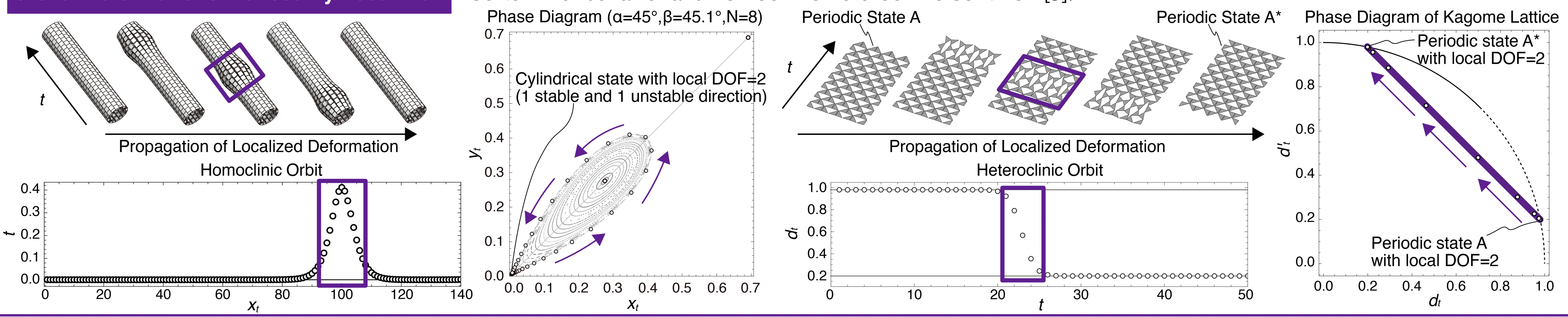
Undulations of rotationally symmetric origami tessellations and **quasiperiodic solutions/conservative systems** [1][2].

- Dynamical system of N-fold symmetric waterbomb tube can have fixed point corresponding to a cylindrical foldede state with its gDOF=2. Around such a fixed point, quasiperiodic solutions exist which induces the undulating folded states, where we can change their "Amplitude" and "Phase" by tuning an initial value $\mathbf{x}_0$ [1].
- This undulation is not limited to waterbomb tube, but the universal phenomenon in a family of N-fold symmetric tubular origami tessellations, which we explained by proving that their dynamical system is conservative. The conservativeness vanishes if the crease pattern includes the scaling [2].



Global Deformations Induced by Local DOF

"Soliton-like" behavior and **homoclinic/heteroclinic soliution** [3].



Future Work

- Realize undulations/soliton and so on in the physical prototypes.
- Consider origami tessellations with no symmetry assumptions, which induces the dynamical systems in a higher dimensional space.
- Connect mathematical properties to mechanical properties; <u>e.g.</u>, A periodic state with the large number of unit cells with its gDOF>0 ⇒ Flexible, and gDOF=0 ⇒ Rigid?
- Can we realize phenomena known in the dynamical systems theory such as **Chaos**?

This work was supported by JSPS KAKENHI Grant Number JP23KJ0682 and JST PREST Grant Number JPMJPR1927.

[1] R. Imada & T. Tachi. "Geometry and Kinematics of Cylindrical Waterbomb Tessellation." ASME Journal of Mechanisms and Robotics, (2022).
[2] R. Imada & T. Tachi. "Undulation in Axisymmetric Tubular Origami Tessellations: a Connection to Area-Preserving Map." Chaos, (2023), (accepted).
[3] R. Imada & T. Tachi. "折紙 / 切紙テセレーションに現れるソリトンと力学系による解析." 日本応用数理学会 2022 年度年会講演予稿集 , (2022).